flame of an oil-flame cannot be altogether deprived of its yellow luminous tip, without serious risk of total extinction; and this faint luminosity is sufficient to prevent pale caps from being seen.

From further experiments made in the above testing chamber with flames produced by alcohol and by hydrogen, it was found to be true in practice, as might be inferred from theory, that, if the flame was pale and practically non-luminous, the size and definition of the flame-cap was augmented by increasing either the size or the temperature of the flame. It is quite possible by attending to these conditions to obtain a flame which, although it is very sensitive for low percentages of gas, becomes unsuitable for the measurement of any proportion of gas exceeding 3 per cent. This must, for the general purposes of the miner, be looked upon as a defect; but it is not a fault of the lamp already referred to. It is of interest to note that with the Pieler spirit-lamp a flame-cap an inch in height was seen in air containing only 0·5 per cent. of methane.

V. "On the Forces, Stresses, and Fluxes of Energy in the Electromagnetic Field." By Oliver Heaviside, F.R.S. Received June 9, 1891.

(Abstract.)

The abstract nature of this paper renders its adequate abstraction difficult. The principle of conservation of energy, when applied to a theory such as Maxwell's, which postulates the definite localisation of energy, takes a more special form, viz., that of the continuity of energy. Its general nature is discussed. The relativity of motion forbids us to go so far as to assume the objectivity of energy, and to identify energy, like matter; hence the expression of the principle is less precise than that of the continuity of matter (as in hydrodynamics), for all we can say in general is that the convergence of the flux of energy equals the rate of increase of the density of the energy; the flux of the energy being made up partly of the mere convection of energy by motion of the matter (or other medium) with which it is associated localisably, and partly of energy which is transferred through the medium in other ways, as by the activity of a stress, for example, not obviously (if at all) representable as the convection of energy. Gravitational energy is the chief difficulty in the way of the carrying out of the principle. It must come from the ether (for where else can it come from?), when it goes to matter; but we are entirely ignorant of the manner of its distribution and transference. But, whenever energy can be localised, the principle of continuity of energy is (in spite of certain drawbacks connected with the circuital flow of energy) a valuable principle which should be utilised to the

uttermost. Practical forms are considered. In the electromagnetic application the flux of energy has a four-fold make-up, viz., the Poynting flux of energy, which occurs whether the medium be stationary or moving; the flux of energy due to the activity of the electromagnetic stress when the medium is moving; the convection of electric and magnetic energy; and the convection of other energy associated with the working of the translational force due to the stress.

As Electro-magnetism swarms with vectors, the proper language for its expression and investigation is the Algebra of Vectors. An account is therefore given of the method employed by the author for some years past. The quaternionic basis is rejected, and the algebra is based upon a few definitions of notation merely. It may be regarded as Quaternions without quaternions, and simplified to the uttermost; or else as being merely a conveniently condensed expression of the Cartesian mathematics, understandable by all who are acquainted with Cartesian methods, and with which the vectorial algebra is made to harmonise. It is confidently recommended as a practical working system.

In continuation thereof, and preliminary to the examination of electromagnetic stresses, the theory of stresses of the general type, that is, rotational, is considered; and also the stress activity, and flux of energy, and its convergence and division into translational, rotational, and distortional parts; all of which, it is pointed out, may be associated with stored potential, kinetic, and wasted energy, at least so far as the mathematics is concerned.

The electromagnetic equations are then introduced, using them in the author's general forms, *i.e.*, an extended form of Maxwell's circuital law, defining electric current in terms of magnetic force, and a companion equation expressing the second circuital law; this method replacing Maxwell's in terms of the vector potential and the electrostatic potential, Maxwell's equations of propagation being found impossible to work and not sufficiently general. The equation of activity is then derived in as general a form as possible, including the effects of impressed forces and intrinsic magnetisation, for a stationary medium which may be eolotropic or not. Application of the principle of continuity of energy then immediately indicates that the flux of energy in the field is represented by the formula first discovered by Poynting. Next, the equation of activity for a moving medium is considered. It does not immediately indicate the flux of energy, and, in fact, several transformations are required before it is brought to a fully significant form, indicating (1) the Poynting flux, the form of which is settled; (2) the convection of electric and magnetic energy; (3) a flux of energy which, from the form in which the velocity of the medium enters, represents the flux of energy due

to a working stress. Like the Poynting flux, it contains vector products. From this flux the stress itself is derived, and the form of translational force, previously tentatively developed, is verified. It is assumed that the medium in its motion carries its properties with it unchanged.

A side matter which is discussed is the proper measure of "true" electric current, in accordance with the continuity of energy. It has a four-fold make-up, viz., the conduction current, displacement current, convection current (or moving electrification), and the curl of the motional magnetic force.

The stress is divisible into an electric and a magnetic stress. These are of the rotational type in eolotropic media. They do not agree with Maxwell's general stresses, though they work down to them in an isotropic homogeneous stationary medium not intrinsically magnetised or electrised, being then the well-known tensions in certain lines with equal lateral pressures.

Another and shorter derivation of the stress is then given, guided by the previous, without developing the expression for the flux of energy. Variations of the properties permittivity and inductivity with the strain can be allowed for. An investigation by Professor H. Hertz is referred to. His stress is not agreed with, and it is pointed out that the assumption by which it is obtained is equivalent to the existence of isotropy, so that its generality is destroyed. The obvious validity of the assumption on which the distortional activity of the stress is calculated is also questioned.

Another form of the stress vector is examined, showing its relation to the fictitious electrification and magnetic current, magnetification and electric current, produced on the boundary of a region by terminating the stress thereupon; and its relation to the theory of action at a distance between the respective matters and currents.

The stress subject is then considered statically. The problem is now perfectly indeterminate, in the absence of a complete experimental knowledge of the strains set up in bodies under electric and magnetic influence. Only the stress in the air outside magnets and conductors can be considered known. Any stress within them may be superadded, without any difference being made in the resultant forces and torques. Several stress formulæ are given, showing a transition from one extreme form to another. A simple example is worked out to illustrate the different ways in which Maxwell's stress and others explain the mechanical actions. Maxwell's stress, which involves a translational force on magnetised matter (even when only inductively magnetised), merely because it is magnetised, leads to a very complicated and unnatural way of explanation. It is argued, independently, that no stress formula should be allowed which indicates a translational force of the kind just mentioned.

Still the matter is left indeterminate from the statical standpoint. From the dynamical standpoint, however, we are led to a certain definite stress distribution, which is also, fortunately, free from the above objection, and is harmonised with the flux of energy. A peculiarity is the way the force on an intrinsic magnet is represented. It is not by force on its poles, nor on its interior, but on its sides, referring to a simple case of uniform longitudinal magnetisation; *i.e.*, it is done by a *quasi*-electromagnetic force on the fictitious electric current which would produce the same distribution of induction as the magnet does. There is also a force where the inductivity varies. This force on fictitious current harmonises with the conclusion previously arrived at by the author that, when impressed forces set up disturbances, such disturbances are determined by the curl of the impressed forces, and proceed from their localities.

In conclusion it is pointed out that the determinateness of the stress rests upon the assumed localisation of the energy and the two laws of circuitation, so that with other distributions of the energy (of the same proper total amounts) other results would follow; but the author has been unable to produce full harmony in any other way than that followed.

VI. "Comparison of Simultaneous Magnetic Disturbances at several Observatories, and Determination of the Value of the Gaussian Functions for those Observatories." By W. GRYLLS ADAMS, D.Sc., F.R.S., Professor of Natural Philosophy in King's College, London. Received June 11, 1891.

(Abstract.)

After drawing attention to previous investigations on this subject, and pointing out the importance of adopting the same scale values for similar instruments at different Observatories, especially at new Observatories which have been recently established, the discussion of special magnetic disturbances is undertaken, especially the disturbances of a great magnetic storm which occurred on June 24 and 25, 1885, for which photographic records have been obtained from 17 different Observatories: 11 in Europe, 1 in Canada, 1 in India, 1 in China, 1 in Java, 1 at Mauritius, and 1 at Melbourne.

The records are discussed and compared, tables are formed of the simultaneous disturbances, and the traces are reduced to Greenwich mean time and brought together on the same plates arranged on the same time-scale. Plates I and II show the remarkable agreement between the disturbances at the different Observatories, and the